PHYS 101 – General Physics I Midterm Exam 1

Duration: 90 minutes | Saturday, 2 March 2024; 13:30

1. Two objects, A and B, are moving in the same direction along a straight track. Object A has speed $v_A = 36\ \mathrm{m/s}$, which is constant (i.e., $a_A = 0$), while object B starts from rest (i.e., $v_{B0} = 0$) and accelerates with acceleration $a_B = 2\ \mathrm{m/s^2}$ until it reaches its maximum speed of $40\ \mathrm{m/s}$, and keeps on going with its constant top speed. They are both at the origin ($x = 0$) at time $t = 0$. **Answer the following questions by writing your answer in the box.**

(a) (5 Pts.) What is the position of object A as a function of time?

Answer: $x_A = 36\,t$ (m)

$$x_A = x_{A0} + v_{A0}t + \frac{1}{2}a_A t^2\,, \qquad x_{A0} = 0\,, \qquad v_{A0} = 36\ \mathrm{m/s}\,, \qquad a_A = 0\,,$$

(b) (5 Pts.) What is the position of object B as a function of time before it reaches its maximum speed?

Answer: $x_B = t^2$ (m)

$$x_B = x_{B0} + v_{B0}t + \frac{1}{2}a_B t^2\,, \qquad x_{B0} = 0, \qquad v_{A0} = 0\,, \qquad a_B = 2\ \mathrm{m/s^2}$$

(c) (5 Pts.) At what time t_m does object B reach its maximum speed?

Answer: $t_m = 20$ (s)

$$v_B = a_B t \quad \rightarrow \quad t_m = \frac{v_{Bm}}{a_B} = \frac{40\ \mathrm{m/s}}{2\ \mathrm{m/s^2}}$$

(d) (5 Pts.) What is the distance d_{AB} between object A and object B when object B reaches its maximum speed?

Answer: $d_{AB}(t_m) = 320$ (m)

$$x_A(20) - x_B(20) = 720 - 400 = 320\ \ (\mathrm{m})$$

(e) (5 Pts.) At what time T does object B catch object A?

Answer: $T = 80 + 20 = 100$ (s)

Object B catches object A after B reaches its maximum speed.

$$x_A(t) = x_B(t) \quad \rightarrow \quad 320 + 36\,t = 40\,t \quad \rightarrow \quad t = 80\ \mathrm{s}$$

(f) (5 Pts.) Where does object B catch object A?

Answer: $x_A(T) = x_B(T) = 3600$ (m)

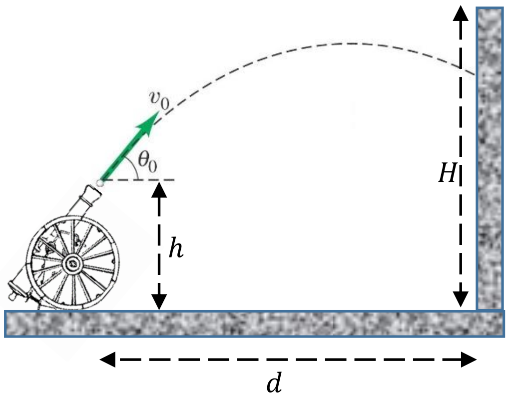
$$x_A(100) = (36\ \mathrm{m/s})(100\ \mathrm{s})$$

2. A projectile is fired from a fixed toy cannon with an initial speed v_0 at $\theta_0 = 45°$ with respect to the horizontal ($\sin\theta_0 = \cos\theta_0 = 1/\sqrt{2}$). Initially, the projectile is at a height h above the ground level, and at a horizontal distance d from a vertical wall of height H. (Use the symbol g for gravitational acceleration. Do not use a numerical value.)

(a) (15 Pts.) What is the minimum v_0 in terms of the parameters H, h, d and the gravitational acceration g if the projectile is to go over the wall?

(b) (10 Pts.)For what range of values of H the projectile can not go over the wall no matter how large its initial speed v_0 is?

(c) (10 Pts.) Now suppose that the cannon is moving horizontally away from the wall with speed u when the projectile is fired, and the projectile does not hit the wall. What is the minimum u for a given v_0?



Solution:

(a) If the projectile is to go over the wall, it must at least be at the point (d, H) at some time. Since $\theta_0 = 45°$, so $\sin\theta_0 = \cos\theta_0 = 1/\sqrt{2}$, we have $v_{x0} = v_{y0} = v_0/\sqrt{2}$.

$$x(t) = v_{x0}t\,, \qquad y(t) = h + v_{y0}t - \frac{1}{2}\text{g}\,t^2\,, \qquad d = v_{x0}t_d \;\rightarrow\; t_d = \frac{d}{v_{x0}}$$

$$y(t_d) = H \;\rightarrow\; H = h + v_{y0}\left(\frac{d}{v_{x0}}\right) - \frac{1}{2}\text{g}\left(\frac{d}{v_{x0}}\right)^2 \;\rightarrow\; H = h + d - \frac{\text{g}d^2}{v_0^2} \;\rightarrow\; v_0 = \sqrt{\frac{\text{g}d^2}{h+d-H}}$$

which is the minimum initial speed required for the projectile to go over the wall.

(b) We note that in the limit $H \rightarrow h + d$, the required initial speed goes to infinity. Therefore, for $H > h + d$ the projectile can not go over the wall starting with finite initial speed.

(c) If the cannon is moving horizontally away from the wall with speed u when the projectile is fired, the horizontal component of projectile's initial veolocity relative to the ground is $v_{x0} - u$. In this case kinematical equations describing the motion of the projectile are

$$x(t) = (v_{x0} - u)t\,, \qquad y(t) = h + v_{y0}t - \frac{1}{2}\text{g}\,t^2\,.$$

The projectile hits the ground at time t_f when $y(t_f) = 0$.

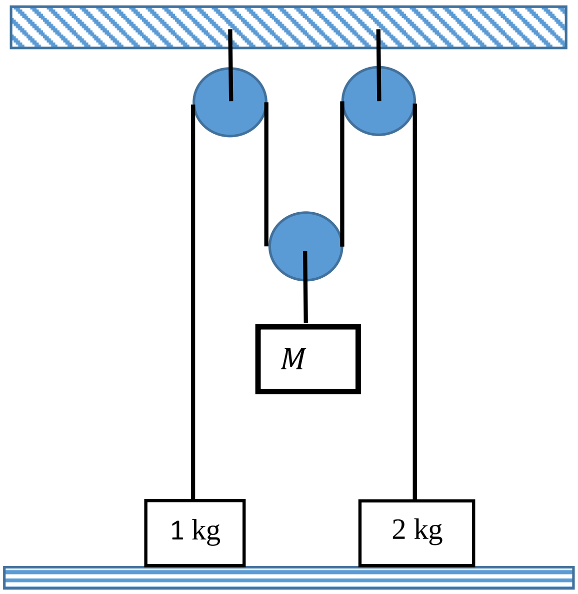
$$h + v_{y0}t_f - \frac{1}{2}\text{g}\,t_f^2 = 0 \;\rightarrow\; t_f = \frac{v_{y0}}{\text{g}} + \sqrt{\left(\frac{v_{y0}}{\text{g}}\right)^2 + \frac{2h}{\text{g}}}\,,$$

where the positive root was taken. For minimum value of u we must have $x(t_f) = d$. Therefore,

$$(v_{x0} - u_{\min})t_f = d \;\rightarrow\; u_{\min} = v_{x0} - \frac{d}{t_f} \;\rightarrow\; u_{\min} = v_{x0} - \frac{\text{g}d}{v_{y0} + \sqrt{v_{y0}^2 + 2\text{g}h}}\,.$$

$$u_{\min} = \frac{1}{\sqrt{2}}\left(v_0 - \frac{2\text{g}d}{v_0 + \sqrt{v_0^2 + 4\text{g}h}}\right).$$

3. Two blocks, with masses 1 kg and 2 kg are resting on the floor. Both are tied to a mass M with the arrangement of pulleys where all ropes are vertical as shown in the figure. The pulleys and the inextensible rope have negligible mass and there is no friction in the system. The masses are released from the configuration shown.



(a) (8 Pts.) What is the minimum value of M that can start motion in the system?

(b) (9 Pts.) What is the minimum value of M so that both the 1 kg and the 2 kg masses are lifted from the floor?

If $M = 3$ kg, and $g = 10\ \mathrm{m/s^2}$:

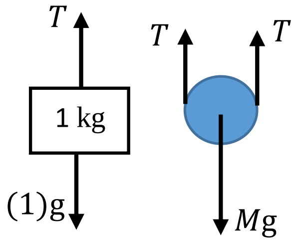
(c) (9 Pts.) Draw free body diagrams for all masses.

(d) (9 Pts.) Find the tension in the rope.

Solution:

(a) For the system to start moving the tension in the rope must at least be able to lift the 1 kg block up the floor. That is $T > (1\text{ kg})g$. Balance of forces on the middle pulley requires $Mg - 2T > 0$, meaning that $M > 2$ kg.

(b) When the system stars moving, 1 kg mass and the pulley in the middle with the mass M will be accelerating. Because the length of the rope is constant, if the 1 kg mass accelerates up with acceleration a, the middle pulley and the mass M will be accelerting down with acceleration $a/2$. Newton's second law implies



$$T - (1\ kg)g = (1\ kg)a\,, \qquad Mg - 2T = M\left(\frac{a}{2}\right) \quad \rightarrow \quad T = a + g\,,$$

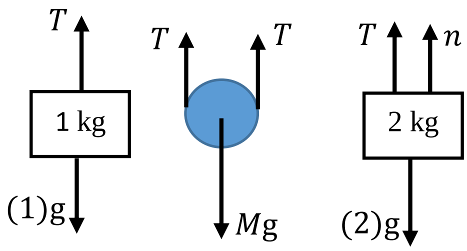
$$a = \frac{2(M-2)}{M+4}g\,, \qquad T = \left(\frac{3M}{M+4}\right)g\,.$$

To lift the two kg mass up the floor the tension must satisfy $T > (2\text{ kg})g$, meaning that

$$\frac{3M}{M+4} > 2 \quad \rightarrow \quad M > 8\text{ kg}\,.$$

(c) (d) For $M = 3$ kg the 2 kg mass will not be accelerating. Since the tension T in the rope is same everywhere, Newton's second law implies

$$T - g = a_1\,, \qquad 3g - 2T = 3a_2\,.$$



Since the length of the rope is constant, we have $a_2 = a_1/2$. Solving these equations for the tension T, we get

$$a_1 = T - g\,, \qquad \frac{a_1}{2} = g - \frac{2}{3}T \quad \rightarrow \quad 2g - \frac{4}{3}T = T - g \quad \rightarrow \quad T = \left(\frac{9}{7}\right)g = \frac{90}{7}\ \text{(N)}.$$